Note on the Single Equation which comprises the Theory of the Fundamental Instruments of the Observatory. By Sir Robert Ball, LL.D., F.R.S.

We may conceive a generalised astronomical instrument of which the essential parts are as follows:—

There is a fundamental axis, which we shall distinguish as axis I. It is capable of rotation in fixed bearings, and to it is attached an index which points to a reading R on a fixed graduated circle A. Axis I passes through the centre of A, and is normal to the plane of A.

Axis II is capable of rotation in bearings fixed on axis I. A second graduated circle B is attached to axis II which passes through the centre of B, and is normal to its plane. The reading of B is R′, as shown by an index rigidly attached to I. It may be observed that an index parallel to the intersections of the planes of A and B will serve for reading both circles, and the geometry of the question is simplified by employing this index.

It is necessary to distinguish between the two poles on the celestial sphere which are defined by the plane of a graduated circle. From one of these poles the graduation would appear to increase clockwise. From the other pole the graduation would appear to increase anti-clockwise. It is the latter pole which we shall here employ. The angle between I and II is the angle ($\ngtr 180°$) between the poles of A and B. We shall express it by $90° - q$.

The telescope is rigidly attached to axis II, and when the optical axis of the telescope is directed to a star, the arc ($\ngtr 180°$) from that star to the pole of B is also a constant of the instrument. We shall denote it by $90° + r$.

The semiplane through axis II and that half of the telescope which contains the objective, cuts B at the graduation we shall term Δ.

Let R_1, R_1' and R_2, R_2' be the readings of the instrument when directed successively to stars S_1 and S_2, with celestial co-ordinates α_1, δ_1 and α_2, δ_2. These co-ordinates may be altitude and azimuth, or right ascension and declination, or latitude and longitude, or any system in which the fundamental circles are rectangular. Then the equation we desire is obtained by equating two different

expressions for the cosine of the arc between S_1 and S_2, and it is as follows:—

$$
\begin{aligned}
&\sin\delta_1 \sin\delta_2 + \cos\delta_1 \cos\delta_2 \cos(\alpha_1 - \alpha_2) \\
&= +\sin^2 q \sin^2 r \\
&\quad + \cos^2 q \sin^2 r \cos(R_1 - R_2) \\
&\quad + \cos^2 q \cos^2 r \sin(R_1' - \Delta) \sin(R_2' - \Delta) \\
&\quad + \cos^2 r \cos(R_1 - R_2) \cos(R_1' - \Delta) \cos(R_2' - \Delta) \qquad \text{(i)} \\
&\quad + \sin^2 q \cos^2 r \cos(R_1 - R_2) \sin(R_1' - \Delta) \sin(R_2' - \Delta) \\
&\quad + \cos^2 r \sin q \sin(R_1 - R_2) \sin(R_1' - R_2') \\
&\quad + \cos q \sin r \cos r \sin(R_1 - R_2) \{\cos(R_2' - \Delta) - \cos(R_1' - \Delta)\} \\
&\quad + \sin q \cos q \sin r \cos r \{\cos(R_1 - R_2) - 1\} \{\sin(R_1' - \Delta) \\
&\qquad\qquad + \sin(R_2' - \Delta)\}
\end{aligned}
$$

By assigning suitable values to q and r, this formula can be made to apply to the following astronomical instruments:—the altazimuth, the meridian circle, the prime vertical instrument, the equatorial, and the almucantar. For the meridian circle q and r should be each as near zero as possible, and for the almucantar q is the latitude and r quite arbitrary. The following general proof will show that the complete theory of each of the instruments named must be included in this one formula.

From any such instrument we demand no more than that the two readings R and R′ obtained by directing the instrument to any particular star shall enable us to calculate the co-ordinates α, δ of that star free from all instrumental errors.

Let S_1, S_2, S_3 be three standard stars of which the co-ordinates are known, and let each of these stars be observed with the generalised instrument with results R_1, R_1'; R_2, R_2'; R_3, R_3' respectively. Substituting for each of the three pairs (S_1 S_2), (S_2 S_3), (S_3 S_1) in the typical formula (i), we obtain three independent equations. From these equations, q, r, and Δ can be found. Nor will there be any indefiniteness in the solution, for in each case we may regard these quantities as approximately known, so that to obtain the accurate values of q, r, and Δ we shall have to solve only linear equations. We may thus regard (i) as an equation connecting α_1 δ_1, α_2 δ_2, R_1, R_1', R_2, R_2', and known quantities.

Let S be the star whose co-ordinates α, δ are sought. We write the equation (i) for the pair (S S_1), and substitute their numerical values for α_1, δ_1, R_1, R_1'. We thus have an equation connecting the co-ordinates α, δ of *any* star with its corresponding R, R′ and known numerical quantities. When we substitute for R and R′ the values observed for S, the formula reduces to a numerical relation between the α and δ of the particular star S. From the pair (S S_2) we find in like manner another quite independent numerical equation involving α, δ. As, however, the equations are not generally sufficient to determine α, δ without indefiniteness, we obtain a third equation from (S S_3). This

equation is not independent of the others, but if we make $x = \sin\delta$, $y = \cos\delta\cos\alpha$, $z = \cos\delta\sin\alpha$, we shall obtain three linear equations in x, y, z which can be solved, and thus α and δ are found without any ambiguity whatever.

All the ordinary formulæ used in connection with the different instruments named can be deduced as particular cases of the general equation (i).

In general, there are no real values of R and R′ when the instrument is directed to the pole of circle A. In such a case R would have to be set on one of the imaginary circular points at infinity.

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The Perturbations of Halley's Comet in the Past. Second Paper. The Apparition of 1222. By P. H. Cowell, M.A., F.R.S., and A. C. D. Crommelin, B.A.

In the first paper of this series we identified the comet of October 1301 with Halley's, and found the value 44″·858 for the mean daily motion at that epoch We have now completed (with the aid of Mr. F. R. Cripps) the calculation of the perturbations by Jupiter and Saturn for the preceding revolution. As a first approximation, Hind's date (mid-July 1223) was assumed for the preceding perihelion passage, and on this assumption the results were as follows :—

Planet.	Limit of u.	dn.	$d\varpi$.	$d\zeta$.
Jupiter	0°– 90°	$-''·2261$	$-357''$	$-6550''$
,,	90–270	$-·0823$	-35	$+6001$
,,	270–360	$+·5624$	-183	-353
Saturn	0– 90	$-·0619$	$+12$	-1784
,,	90–270	$+·0636$	-39	$+1602$
,,	270–360	$-·2411$	$+15$	$+41$
Sum	...	$+·0146$	-587	-1043

Hence mean motion in $1223 = 44''·858 - 0''·015 = 44''·843$

$$\text{and calculated period} = \frac{1296000'' + 1043''}{44''·843} = 28924 \text{ days.}$$

This indicated 1222 August 15 as the date of the preceding perihelion passage, or 11 months earlier than Hind's date. This is too large a discordance to be possible, so Hind's identification of